

Orthogonal electrocardiogram derived from the limb and chest electrodes of the conventional 12-lead system

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Abstract—Methods of computing orthogonal X, Y and Z voltages representing the equivalent heart dipole activity are described. These components were obtained as linear functions of voltages from the limb and chest electrodes of the conventional 12-lead system. The coefficients of these linear functions are given and a close relationship with the Frank lead system is demonstrated. Two methods were used to obtain transformation coefficients—lead vector approach and statistical least-squares fit. ECG data from 92 individuals (healthy and with cardiac disorders) were used to obtain adequate coefficient values.

Keywords—Electrocardiography, Lead system, Least-squares fit, Multichannel amplifier, Signal comparison, Vectorcardiography

Med. & Biol. Eng. & Comput., 1987, 25, 155–164

1 Introduction

TWO ELECTROCARDIOGRAPHIC systems (Fig. 1) are the most popular nowadays—the conventional 12-lead system and the orthogonal system of Frank. The conventional system is widely accepted and probably over 90 per cent of the world's ECG recordings are made with this system. On the other hand, the so-called corrected orthogonal lead systems (such as the Frank system) have a solid biophysical background and, along with the additional empirically accumulated diagnostic information, give us the opportunity to make more direct correlations between the electrocardiogram and the electrical activity of the heart. The orthogonal systems are not widely used at present. Some of the reasons are of technical origin: another set of electrodes, different equipment etc. The goal of this investigation is the synthesis of an orthogonal system that uses the same electrode positions (limb and chest) as the conventional 12-lead system and at the same time is close enough to the orthogonal system of Frank to apply existing diagnostic criteria and rules.

1.1 Relation to previous work

FRANK (1956) proposed the well known orthogonal lead system. The three components (X, Y, Z) were computed as linear functions of voltages from a special set of electrodes. His results were based on experiments with a homogeneous human torso model and on the assumption that the electrical activity of the heart can be represented as a stationary dipole.

Several difficulties arise when attempting to design a clinically practical orthogonal lead system. The assumption of a stationary dipole is only an approximation of the real heart excitation process. In fact, the dipole location is

not fixed and moves with different phases of heart excitation. Frank has dealt with this problem by appropriately

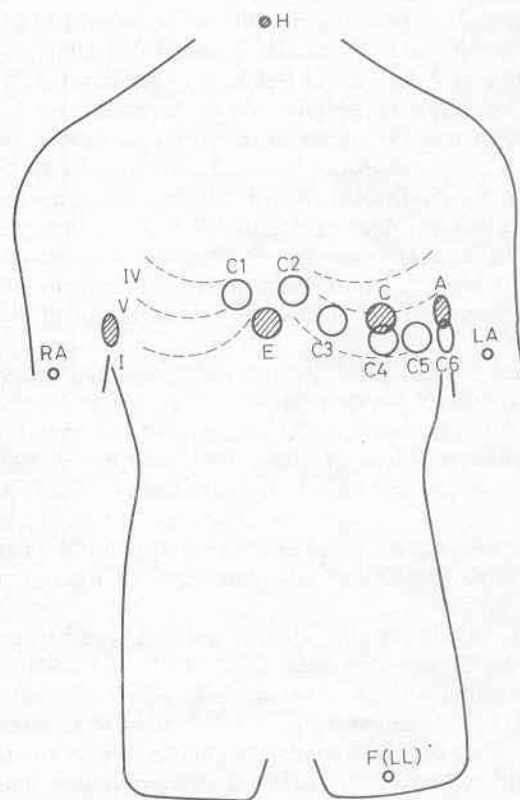


Fig. 1 Electrode locations of the conventional 12-lead system and the Frank orthogonal lead system. Frank electrodes are shaded. The chest electrodes are deliberately enlarged to show the uncertainty of their placement on the human torso. All voltages are referred to the left leg electrode F. The Frank M electrode is placed on the back of the patient on the projection point of the E electrode

First received 14th January 1985 and in final form 20th May 1986

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Appendix 1

Relationship between standard lead voltages (II, III, V1 . . . , V6) and those used by the author. Substitution can be made and transformation coefficients with standard voltages can be obtained.

$$\begin{aligned} RA &= -II & LA &= -III \\ Ci &= Vi - (II + III)/3 \end{aligned} \quad (13)$$

Appendix 2

Analogue computation of the orthogonal voltages

In our investigation the signals X_C , Y_C and Z_C were computed by a microprocessor system. These signals can also be calculated by means of a resistive network, as is the case with the classical orthogonal lead systems. Fig. 6 shows the circuit diagram of a

simple device that computes the orthogonal components according to eqn. 12. The same electrodes are used simultaneously for conventional electrocardiographic recording.

It is necessary to use an eight-channel ECG amplifier to compute the orthogonal components. A simple circuit (LEVKOV, 1982) is used and the classical differential instrumentation amplifiers are avoided. Body potential driving is applied and the mains interference is reduced to an acceptable level.

The eight buffer amplifiers have a gain of 10. In this way the signal level is raised and the signal-to-noise ratio cannot be impaired by the resistive network. Signals with normal levels are applied to the input amplifiers of the 12-lead electrocardiograph through the patient cable. The left leg (LL') and the right leg (RL') inputs of the electrocardiograph are connected to the ground. The differential symmetry of the amplifiers is affected but the noise immunity is not impaired due to the low impedance in these points. The output orthogonal voltages X_C , Y_C and Z_C can be fed to a suitable recording device such as a vectorcardiograph, plotter etc.

Author's biography



Chavdar L. Levkov was born in Sofia, Bulgaria in 1949. He obtained his BS degree in Electronics from the Faculty of Radioelectronics, Sofia, in 1973. He has been a research engineer at the Institute of Medical Engineering of the Medical Academy since 1975, working on analogue preprocessing and computing circuits for ECG analysis. His current interests include hardware and software problems of modern electrocardiography as well as biophysical phenomena involved with heart excitation processes.

selecting the electrode sites and the linear functions of their voltages. Thus, the so-called lead vectors of the three orthogonal components become insensitive to dipole wandering within the region of the heart silhouette.

Frank published data for lead vectors of multiple electrodes placed on the surface of the homogeneous torso (FRANK, 1954). These data have been widely used in our investigation. In the 1950s, numerous orthogonal lead systems were proposed and, along with the conventional lead system, a practical and theoretical interest arose in possible transformations between them. BURGER *et al.* (1959; 1962) have shown the possibility of transforming one orthogonal lead system into another by means of linear transformation. The coefficients of transformation have been obtained by applying the least-squares (LS) fit method. The accuracy of the transformation has been assessed quantitatively on the basis of a population group of 169 individuals. They have concluded that 'well-founded orthogonal systems might be transformed into each other with satisfactory results.

Approximately at the same time, PIPBERGER *et al.* (1961) in a clinical investigation on 261 patients concluded that 'informational content of the three corrected orthogonal leads is comparable to that of the standard 12 lead ECG' and that resolved (i.e. rotated by linear transformation) orthogonal leads have waveform patterns that correlate strongly to corresponding standard precordial leads. No quantitative results have been given.

HORAN *et al.* (1965) have also investigated the possibility of transformation between different orthogonal lead systems. The LS fit method was used, but the average transformation coefficients were obtained by averaging individual coefficients (see method 2a). 35 healthy individuals were recorded and the conclusion was that 'practical interchangeability of quantitative information obtained from one lead system into that obtainable by another is seriously limited because of the wide range of biological variation in transformation characteristics'.

Another approach has been used by Dower and co-authors (DOWER, 1971; DOWER *et al.*, 1980; DOWER, 1984). They have used Frank lead vector data and computed linear transformation coefficients to obtain the conventional 12 ECG signals from X, Y, Z signals of the Frank system: the so-called derived ECG. A widespread clinical investigation has been carried out and satisfying results have been declared with the conclusion that such a transformation is very convenient for clinical purposes. No quantitative results of transformation accuracy have been given.

WOLF *et al.* (1976) have investigated the two possibilities of transformations: from Frank to conventional and vice versa. For the first transformation LS fit and Dower methods have been tested. Only the LS fit method has been used for derivation of Frank leads from conventional. Quantitative results have been obtained for a group of 113 individuals. The conclusions are that there is a substantial difference between the original and the synthesised leads; the LS fit method is better than Dower's method; the derivation of Frank leads from conventional works much better than the reverse—'synthesised Frank leads can be considered acceptable if only QRS and ST-T interpretation is considered'. Unfortunately, they have not published the coefficient values obtained by the LS fit method.

Kornreich and co-authors (KORNREICH, 1973; KORNREICH and RAUTAHARJU, 1981) have synthesised the voltage obtained from an electrode arbitrarily placed on the torso surface by means of linear functions of voltages obtained either from the conventional electrodes or from the Frank electrodes. They have found that there are zones

on the torso for which such a procedure is not successful. An interesting conclusion can be drawn when analysing their experimental results: the routine 12-lead and the Frank lead system are very similar as far as their information content is concerned. Hence, mutual transformation is theoretically possible.

As can be seen, in different investigations controversial opinions can be met and general agreement is not reached. But practical needs for simultaneous conventional and orthogonal ECG information still exist and this has led some authors to use both systems, accepting the inconveniences of placing two electrode sets. As a compromise, MACFARLANE *et al.* (1984) designed the so-called hybrid lead system that uses conventional electrodes plus two more electrodes to obtain orthogonal ECG voltages.

2 Instrumentation and methods

Two recordings for every patient are stored—voltages from the conventional and from the Frank electrodes. Each recording consists of eight signals. Eight linearly independent conventional signals LA, RA, C1, ..., C6 between the corresponding electrodes and the left leg electrode make up the conventional system recording. Six linearly independent signals of the Frank system (voltage differences between the well known Frank electrodes and a left leg electrode) A, M, C, H, I, E plus LA and RA make up the other recording. The Frank electrodes are placed at the level of the fifth intercostal space. No special efforts are made to acquire a high degree of electrode placement accuracy as we have tried to simulate everyday clinical practice. Stainless-steel electrodes with a diameter of 25 mm are employed.

An eight-channel ECG amplifier is used with a noise of 0.5 μV (RMS referred to the input) and a frequency response of 0.05–100 Hz (–3 dB level). The signals are digitised at a sampling rate of 250 s^{-1} by an A-D converter with a resolution of 20 μV referred to the input. All signals are stored in the microprocessor system memory and saved on a floppy disk for later processing.

The conventional and the Frank recordings are made successively and LA and RA signals in both recordings are used for synchronisation purposes. The Frank signals X, Y and Z are derived by the microcomputer, avoiding the necessity of using a resistive network for analogue computation.

A program package was designed for ECG processing based on the Forth language. Various procedures, such as filtration, linear combinations, ECG wave recognition, LS fit etc., can be implemented easily in dialogue with the computer. ECG waveforms can be edited visually and drawn on a plotter if necessary.

A simple microcomputer system based on the 8-bit 6800 processor is used to implement the procedures described above.

ECG recordings of 92 individuals were conducted. There were 67 males and 25 females aged between 15 and 69 (mean value 42). In 58 of them there was no evidence of heart disease. The individuals with heart disease (34 cases) were patients of a cardiological clinic and various pathologies were encountered: hypertrophy, bundle branch blocks, myocardial infarction etc.

Additional ECG recordings (Frank and conventional) were made from 19 randomly chosen individuals from the same group in a state of deep inspiration.

Later, the Frank and the computed signals will be denoted with indices 'F' and 'C', respectively.

Two methods were used to obtain transformation

coefficients—the lead vector approach and the least-squares fit.

2.1 Lead vector method (method 1)

The model widely accepted in clinical electrocardiography is that of a stationary heart dipole. The voltage V across two electrodes, according to this model, can be represented as follows:

$$V = L_x P_x + L_y P_y + L_z P_z \quad (1)$$

where L_x , L_y and L_z are the components of the so-called lead vector L , and P_x , P_y and P_z are the components of the so-called heart vector P . If X , Y , and Z voltages proportional to corresponding components of P are used, then the components of L become nondimensional and eqn. 1 can be transformed into

$$V = l_x X + l_y Y + l_z Z \quad (2)$$

The lead vector components l_x , l_y , l_z are scaled according to FRANK (1956).

Theoretically, data from any three linearly independent lead vectors are sufficient to compute the three components of the heart vector. Three equations according to eqn. 2 using, for example, LA , RA and $C1$, can be worked out and the horizontal voltage X can be obtained when this linear system is solved:

$$X = a_1 LA + a_2 RA + a_3 C1 \quad (3)$$

The same equations can be written for Y and Z . This approach has two shortcomings:

- (i) Lead vector values are experimentally obtained from a torso model and considerable discrepancies can be observed between individuals.
- (ii) Lead vector values are very sensitive to heart dipole location even in the region of the heart silhouette.

These disadvantages can occasionally be overcome by proper selection of electrode locations and weighted averaging of the electrode voltages as is the case with all well known corrected orthogonal lead systems.

In our case we cannot choose the electrode locations, so another approach must be used to solve the problem. With the eight lead vectors of the conventional system, 56 linearly independent equations, according to eqn. 3, can be worked out for each orthogonal voltage. For the X voltage

$$\begin{aligned} X(1) &= a_{11} LA + a_{21} RA + a_{31} C1 \\ X(2) &= a_{12} LA + a_{22} RA + a_{32} C2 \\ &\vdots \\ X(56) &= a_{156} C4 + a_{256} C5 + a_{356} C6 \end{aligned} \quad (4)$$

Each linear function is denoted as $X(i)$. A weighted averaging can be done:

$$X_C = \frac{\sum_i w_i X(i)}{\sum_i w_i} \quad (5)$$

where w_i are different arbitrarily chosen weighting factors. The new averaged linear function for X will have eight terms:

$$X_C = a_1 LA + a_2 RA + a_3 C1 + a_4 C2 + a_5 C3 + a_6 C4 + a_7 C5 + a_8 C6 \quad (6)$$

It is obvious that the problem does not have only one solution. That depends upon the weights—the new weights will give new coefficients a_i . In the case where the individual lead vectors are used and if they are insensitive to

dipole location, any linear function will give the same result—the true component of the heart vector. However, in real cases, where fixed lead vectors are used, substantial discrepancies in the results are found between the functions. Proper weights must be chosen to reach the desired result, which is a close relationship with the Frank lead system.

A mathematical consideration can be pointed out which limits the possible values of coefficients in eqn. 6. These coefficients must be as small as possible and preferably less than one. Large values could make the corresponding components very sensitive to individual patient lead vectors and the result will be poor, regardless of the theoretical compensation. The same holds true for eqns. 4 from which eqn. 6 is derived. The equations with large coefficients a_{ij} in eqns. 4 are omitted in the summation equation (eqn. 5).

All lead vector values are taken from Frank's paper (FRANK, 1954). In the paper the locations of the conventional precordial electrodes are not clearly defined. The data are graphically represented on the so-called 'image surface'. Several possible values for each lead vector are considered and their influence on the quality of the transformation is assessed. Lead vectors presented by DOWER *et al.* (1980) were also tested.

2.2 Least-squares fit method (methods 2a and 2b)

The classical LS fit method was implemented in order to obtain the coefficients of transformation that will minimise the mean-squares error between Frank and derived signals. The so-called normal equations must be formed for each component separately. Let us take the X -component:

$$\left. \begin{aligned} \sum_i X_{Fi} LA_i &= a_1 \sum_i LA_i^2 + a_2 \sum_i RA_i LA_i \\ &+ \dots + a_8 \sum_i C6_i LA_i \\ \sum_i X_{Fi} C6_i &= a_1 \sum_i LA_i C6_i + a_2 \sum_i RA_i C6_i \\ &+ \dots + a_8 \sum_i C6_i^2 \end{aligned} \right\} \quad (7)$$

The index i is the successive sample of the digitised ECG signal. The coefficients a_1 – a_8 are obtained after solving this linear system and the best fit transformation has the same form as eqn. 6.

The average (or global) transformation coefficients for a group of individuals can be calculated in two ways:

- (a) Method 2a. The LS fit is performed for each patient separately in a selected time interval (one QRS complex, for example). Individual coefficients are obtained and the global transformation is calculated as a mean value of the individual coefficients.
- (b) Method 2b. The LS fit is performed for multiple recordings simultaneously. All these recordings are assumed to be one large data set. The average transformation coefficients are calculated from the cumulative normal equation (eqn. 7).

The software was developed in such a way as to produce the results in convenient form. For every successive record, individual coefficients and current coefficients according to methods 2a and 2b are computed. This permits one to check continuously the variability of the transformation coefficients.

2.3 Signal comparison

To compare computed with real Frank signals, both represented in discrete form, three quantities are introduced: the distance d , the correlation coefficient r and the regression coefficient b , where for the X component:

$$d_x = \left(\frac{\sum_i (X_{Fi} - X_{Ci})^2}{\sum_i X_{Fi}^2} \right)^{1/2} \quad (8)$$

$$r_x = \frac{\sum_i X_{Fi} X_{Ci}}{\left(\sum_i X_{Fi}^2 \sum_i X_{Ci}^2 \right)^{1/2}} \quad (9)$$

$$b_x = \frac{\sum_i X_{Fi} X_{Ci}}{\sum_i X_{Fi}^2} \quad (10)$$

The same quantities are defined for Y and Z components.

The three-dimensional distance $d3$ is defined as

$$d3 = \left(\frac{\sum_i [(X_{Fi} - X_{Ci})^2 + (Y_{Fi} - Y_{Ci})^2 + (Z_{Fi} - Z_{Ci})^2]}{\sum_i (X_{Fi}^2 + Y_{Fi}^2 + Z_{Fi}^2)} \right)^{1/2} \quad (11)$$

The distance d gives a normalised estimation of the differences between two signals and is a relative measure of the mean-squares error. The correlation coefficient r estimates similarities between two wave patterns and is insensitive to their amplitude differences. The regression coefficient b is a suitable measure of amplitude differences when $r > 0.9$. For identical signals $d = 0$, $r = 1$, $b = 1$.

2.4 Signal processing

The ECG signals from the conventional and Frank leads must be isophasic, otherwise the procedures of fitting and comparing are meaningless. The term isophasic means that all points with the same index in all leads must belong to the same phase of cardiac activity. Any time shift will introduce an error. In our case, where the recordings of the conventional and Frank signals were made at different times, initially identical beats were chosen and then a time synchronisation procedure was applied. LA and RA signals in both recordings were used for this purpose. A simple linear interpolation technique was employed for shifting signals for time intervals smaller than the sampling interval. The offset between electrical zero and the isoelectric level (measured in P-R interval) was removed in all signals. The 50 Hz interference was removed by a pro-

cedure described elsewhere (LEVKOV *et al.*, 1984). The representative P-QRS-T complex was chosen visually for minimum drift and noise.

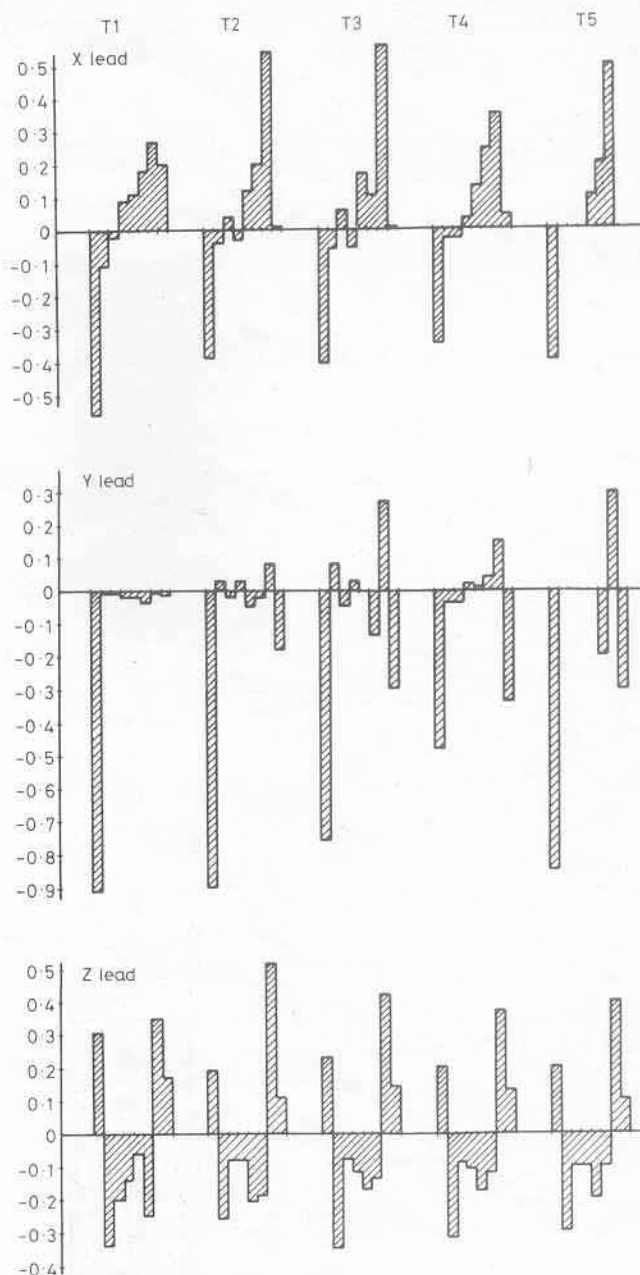


Fig. 2 Graphic representation of transformation coefficients as coefficient patterns. In each pattern the lead weights are in the following order (from left to right): RA, C1, . . . C6, LA

Table 1 Coefficient values for the different transformations tested

	LA	RA	C1	C2	C3	C4	C5	C6	
X	0.20	-0.56	-0.11	-0.02	0.09	0.11	0.18	0.27	T1
	0.00	-0.39	-0.04	0.04	-0.03	0.12	0.15	0.54	T2
	0.00	-0.41	-0.06	0.06	-0.06	0.17	0.10	0.56	T3
	0.04	-0.35	-0.03	-0.03	0.03	0.13	0.24	0.35	T4
Y	-0.01	-0.91	-0.01	-0.01	-0.02	-0.02	-0.04	0.00	T1
	-0.18	-0.90	0.03	-0.02	0.03	-0.05	-0.02	0.08	T2
	-0.30	-0.76	0.08	-0.05	0.03	0.00	-0.14	0.27	T3
	-0.39	-0.48	-0.04	-0.04	0.02	0.01	0.04	0.15	T4
Z	0.17	0.31	-0.34	-0.20	-0.14	-0.06	-0.25	0.35	T1
	0.11	0.19	-0.26	-0.08	-0.08	-0.21	-0.19	0.52	T2
	0.14	0.23	-0.35	-0.08	-0.12	-0.17	-0.14	0.42	T3
	0.13	0.20	-0.32	-0.09	-0.11	-0.18	-0.12	0.37	T4

T1 is obtained by the lead vector method 1. T2 is obtained by the LS fit method 2b on a group of 46 records. T3 is obtained by the LS fit method 2b on the whole group of 92 records. T4 is obtained by the LS fit method 2a on the whole group.

3 Results

There was no evidence of any apparent correlation between the quality of the transformation and the type of disease; therefore the patients were not divided into nosological groups.

3.1 Coefficient values

Method 1 was used at the beginning, when only recordings from 46 individuals were available. Several transformations for each orthogonal component were tested on this smaller group. Different possible values of lead vectors including those presented by DOWER *et al.* (1980), as well as different weights according to eqn. 7, were used to calculate the transformation coefficients. The most trivial case was where all weights $w = 1$ and satisfactory results for X

and Z components were obtained. The Y component computation was much better when the weight for the RA voltage was prevailing. If the mean regression coefficient b was significantly different from 1, the transformation was corrected by division of the transformation coefficients by b . The best results were obtained by the transformation denoted by T1 (first line of Table 1).

Transformations obtained by the LS fit methods are presented in Table 1. T2 was obtained by method 2b applied to the first 46 records; T3 was obtained by method 2b and T4 was obtained by method 2a, both applied to the whole group. The LS fit was performed only on the QRS interval. The coefficients of each transformation are presented graphically in Fig. 2. I call these diagrams 'coefficient patterns' and they prove to be a useful graphic tool when a large number of transformations are tested and a

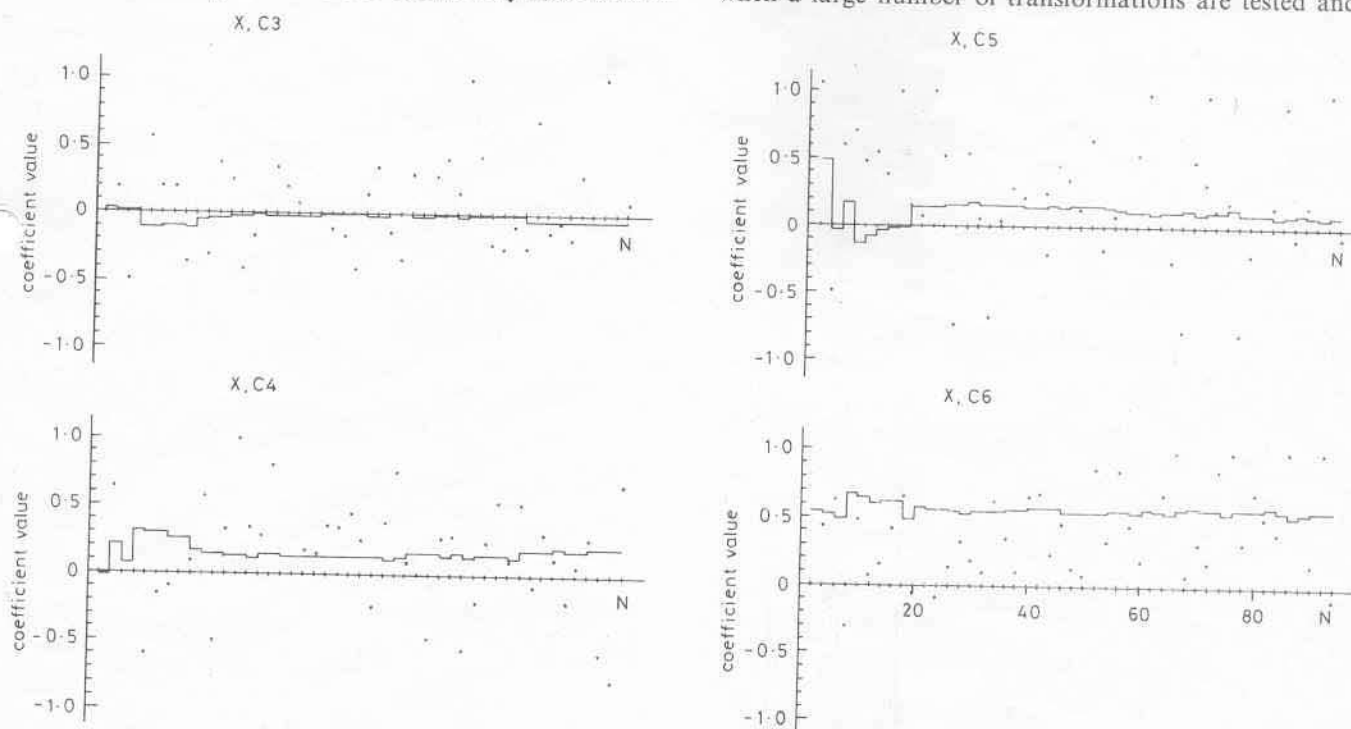


Fig. 3 Variation of some coefficient values obtained by the least-squares fit method for the X component. N is the number of patients. All 92 records are processed. The solid lines are current values obtained by method 2b. The individual coefficients are marked by dots and those with larger values are clipped to ± 1 .

Table 2 Statistical evaluation of different transformations for all 92 cases

	d	SD	per cent	r	SD	per cent	b	SD	
X	0.20	0.09	97	0.991	0.01	100	0.94	0.14	T1
	0.15	0.11	98	0.987	0.05	99	1.02	0.15	T2
	0.15	0.10	98	0.989	0.04	99	1.02	0.12	T3
	0.16	0.12	98	0.986	0.06	99	1.05	0.17	T4
	0.15	0.12	98	0.987	0.06	99	1.04	0.15	T5
Y	0.22	0.16	89	0.970	0.09	92	1.04	0.17	T1
	0.21	0.11	92	0.979	0.03	96	0.96	0.11	T2
	0.19	0.11	93	0.982	0.02	97	1.02	0.13	T3
	0.23	0.15	91	0.976	0.04	95	1.03	0.17	T4
	0.21	0.12	92	0.981	0.03	97	1.00	0.14	T5
Z	0.44	0.33	55	0.940	0.10	84	0.85	0.26	T1
	0.36	0.21	69	0.952	0.06	90	1.08	0.34	T2
	0.35	0.20	74	0.953	0.07	91	1.02	0.27	T3
	0.35	0.19	73	0.952	0.07	89	1.06	0.28	T4
	0.36	0.20	71	0.949	0.08	89	1.05	0.28	T5
d3	0.24	0.08	79						T1
	0.21	0.07	88						T2
	0.19	0.06	96						T3
	0.21	0.06	90						T4
	0.20	0.06	90						T5

T5 is the simplified transformation according to eqn. 12. The percentage of cases with a reasonably good synthesis ($d < 0.4$; $d3 < 0.3$; $r > 0.9$) is given.

quick visual comparison has to be made.

The stability of coefficient values obtained by the LS fit methods with respect to patient-to-patient variation was also checked. As an example, a graphical representation of coefficient values for the X component (method 2b) is given in Fig. 3. After a new recording is added for processing, new coefficients are computed for the new larger data set by method 2b. Individual coefficient values of each new patient are marked with dots. N is the patient number.

3.2 Estimation of the transformation performance

Table 2 presents the mean values for all 92 cases of d , r and b quantities along with their standard deviations (SD) for the different transformations tested. The percentage of cases with reasonably good synthesis ($d < 0.4$; $d3 < 0.3$; $r > 0.9$) is also given. Table 3 presents the results of T2 transformation where the first 46 recordings are assumed to be a training set and the remaining 46 recordings are assumed to be a test set. Table 4 presents the results of T3 transformation on the recordings made in the state of deep inspiration.

Differences between signals which are important from a diagnostic point of view might not yield essential changes in d , r or b . For example, small Q waves are very informative about some cardiac disorders, but their weight in such an integral quantity as d is too small. For this reason the following signals were plotted for every case under question for additional visual inspection:

- 12 conventional signals
- original Frank (X_F , Y_F , Z_F) signals
- derived (X_C , Y_C , Z_C) signals
- frontal, horizontal and left sagittal vectorcardiographic loops
- spatial magnitude signals R_F and R_C , where

$$R = (X^2 + Y^2 + Z^2)^{1/2}$$
- spatial velocity signals V_F and V_C , where

$$V = (dX/dt)^2 + (dY/dt)^2 + (dZ/dt)^2)^{1/2}$$
- differences between corresponding left (LD) and right (RD) arm signals of conventional and Frank recordings (with a double gain).

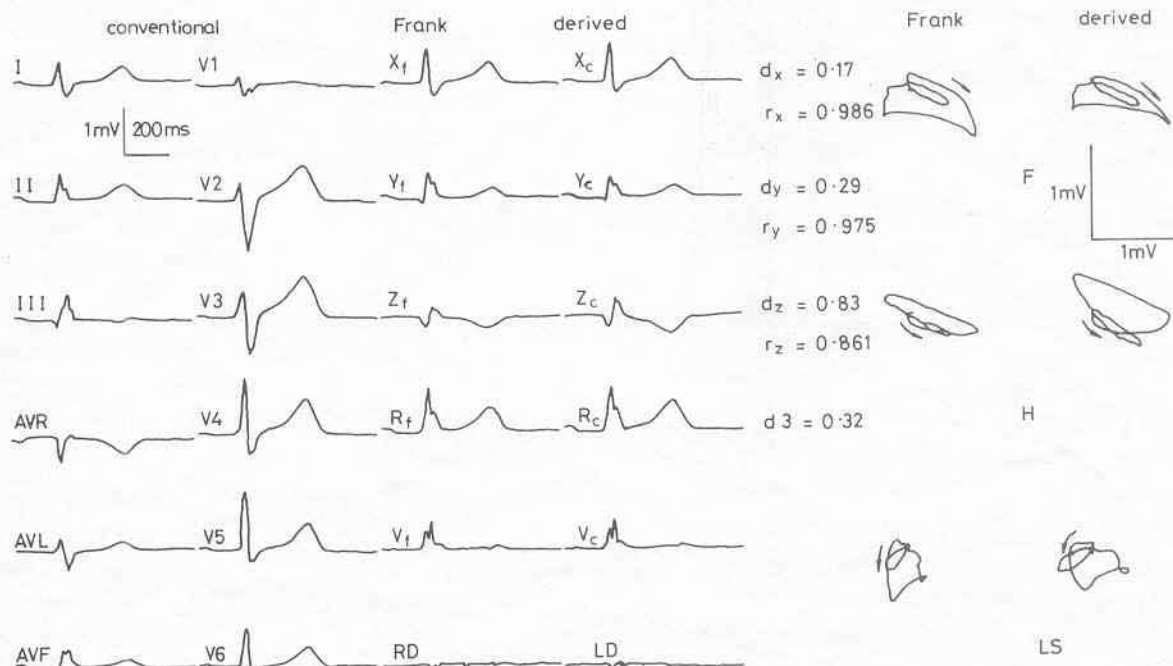


Fig. 4 Example of a recording of a patient with one of the worst syntheses (T3 transformation). The RD and LD curves show that some difference between representative ECG beats of Frank and conventional records exists. RD and LD curves are plotted with a double gain.

Table 3 Evaluation of T2 transformation with training and test sets each containing 46 individuals

	d	SD	r	SD	b	SD
X train.	0.14	0.07	0.993	0.01	1.02	0.11
X test	0.16	0.14	0.983	0.06	1.06	0.16
Y train.	0.21	0.13	0.974	0.04	1.01	0.12
Y test	0.20	0.10	0.983	0.02	0.95	0.13
Z train.	0.36	0.22	0.948	0.08	1.01	0.26
Z test	0.38	0.18	0.954	0.05	1.10	0.30
$d3$ train.	0.19	0.06				
$d3$ test	0.21	0.07				

Table 4 T3 transformation performance on the group of 19 individuals in the state of deep inspiration

	d	SD	r	SD	b	SD
X	0.18	0.10	0.979	0.03	1.05	0.07
Y	0.24	0.11	0.957	0.07	0.99	0.10
Z	0.25	0.10	0.974	0.02	1.05	0.17
$d3$	0.21	0.10				

As an example, one of the cases with the worst synthesis (T3 transformation) is presented in Fig. 4.

3.3 Simplified transformation

A simplified transformation (T5) was derived from transformation T3. The coefficients with small values were omitted. The other coefficients were cut to only one significant digit with values near to the corresponding coefficients of T3. Several possible 'one significant digit' transformations were tested and the following one is almost as good as T3:

$$\left. \begin{aligned} X_C &= -0.4RA + 0.1C4 + 0.2C5 + 0.5C6 \\ Y_C &= -0.3LA - 0.8RA - 0.2C5 + 0.3C6 \\ Z_C &= 0.1LA + 0.2RA - 0.3C1 - 0.1C2 \\ &\quad - 0.1C3 - 0.2C4 - 0.1C5 + 0.4C6 \end{aligned} \right\} \quad (12)$$

The estimation of this T5 transformation is given on the last line in Table 2 and its coefficient pattern in Fig. 3. In

Fig. 5 both Frank and derived from T5 waveforms are presented in compressed form. All 92 records are included.

4 Discussion

4.1 Limitation factors

Frank and conventional ECG records are not recorded simultaneously and different heart beats are compared. I

have carefully chosen almost identical beats for comparison using left arm and right arm signals from both recordings as a reference. A small difference always exists (see Fig. 4) due to beat-to-beat variation which is mainly due to breathing. As an example the mean correlation coefficient (all 92 cases) of RA signals between corresponding Frank and conventional recordings is 0.991 and d is 0.12.



Fig. 5 A compressed representation of Frank and derived (T5) ECG waveforms. All 92 records are included. The records marked with dots are reduced in size by a factor of 2

4.2 Methods

The lead vector method (method 1) was tried first. Promising results were obtained. Averaging of lead vector components substantially reduces the influence of individual variation. The transformations obtained by this method strongly depend upon lead vectors used. DOWER *et al.* (1980) lead vectors were also tested with comparable results, only the Z component computation being slightly worse. A possible explanation is that Dower used a modified Frank system (electrodes on IV intercostal space) to derive his lead vectors.

A good physical model of the problem can be very helpful in interpreting the results when the LS fit method is used. The lead vector approach can be used as a useful frame. The LS fit method is very sensitive to non-homogeneity of input data and a careful examination of input ECG data is needed to eliminate random peaks. When the redundancy and correlation of input data are high, the solution obtained by this method might be mathematically correct but physically impractical. A good example is the computation of individual coefficients. As can be seen from Fig. 3 the dispersion is very high. A possible explanation is that to fit one waveform from eight there are too many possibilities and the actual LS fit solution is determined by random factors (noise in ECG waveforms) rather than by physical limitations. This is probably the reason why method 2a cannot yield as good fitting as method 2b.

Method 2b gives minimum LS error for the entire group. It is important to check the stability of the coefficients to see whether a reliable and physically correct solution is reached. As can be seen from Fig. 3 the coefficient values reach a 'steady state' after the first 20 recordings but after that there are always small fluctuations and drift. In my opinion, when a steady state is reached, one of the numerous possible solutions to the problem is obtained. The fluctuations that are observed do not mean that a much larger group is needed to achieve statistical stability. We just obtain other possible general solutions with minimal LS errors for the particular group of individuals. If we use the coefficient values obtained somewhere in the region of the steady state, we will observe that the results of this intermediate transformation are comparable to the transformations derived from the whole group and its coefficient pattern is similar to the others (see Fig. 2). As an example the intermediate transformation T2 is given on line 2 in Table 1. The results presented in Tables 2 and 3 confirmed quantitatively the above reasoning.

4.3 Optimal transformation

There is probably no optimal (or unique) transformation. If the lead vector theory is assumed (eqn. 6), numerous solutions to the problem exist, at least theoretically. The data of Table 2 show that transformations with significantly different coefficients lead to end results with comparable accuracy. The most important limitation factors for increasing the transformation accuracy are the individual anatomy and the electrode placement. These two factors are deliberately combined into one. The relatively constant spatial configuration between the ventricular mass centre and the electrodes is the desired optimum that will reduce the ECG variability. At the moment any existing procedure aimed at solving this problem will be time-consuming and totally impractical for clinical purposes. Careful electrode placement on well defined anatomical points (RAUTAHARJU, 1976) might be useful for the reduction of variability, especially for sequential records, but the problem of the positioning of the electrodes relative to the heart cannot be avoided.

The non-dipolar heart activity components also limit the accuracy of reconstruction. Non-dipolar local signal components in any of the conventional ECG leads might not be present in Frank leads and vice versa.

4.4 Transformation accuracy

Table 2 presents quantitative results for the transformation performance. The X component computation is probably near the maximum of the practically possible accuracy, reaching that of the beat-to-beat variability. The Y component also has a good accuracy and future improvement will be difficult due to the lack of leads with a stable Y component. Only the RA lead meets such a requirement. The LA lead has extremely variable characteristics between different individuals and the precordial Y components are very sensitive to electrode placement. Probably the Z component transformation might be improved, but not very significantly. This is, possibly, due to proximity effects and lack of an electrode on the back of the patient.

4.5 Coefficient patterns

In view of the coefficient variation obtained by the LS fit method, we can expect that a different group of individuals will give a different set of transformation coefficients. We cannot predict the ranges of variability but, most probably, any new adequate transformation will have a coefficient pattern that resembles the patterns of the whole family of transformations for a given orthogonal component (Fig. 2).

4.6 T-wave

The fitting and estimation of the transformation was performed only in the QRS area. The same procedures were carried out for the entire QRS-T area. While there was some improvement in the T-wave region, certain degradation of the synthesis in the QRS interval was observed. It was decided to fit and estimate only in the QRS interval, which is much more important for clinical diagnosis. The quality of T-wave reconstruction was always good if the QRS reconstruction was successful (see Fig. 5).

4.7 Simplified transformation

There are several considerations, some of them controversial, that have to be taken into account when a simplified transformation is designed.

- Lead signals weakly correlated with the synthesised component, i.e. with small coefficients, can be omitted
- It is better to use more leads to compute a component because sensitivity to electrode placement errors is reduced as well as noise, due to the averaging effect
- There is no unique transformation so it might be expected that some other combination of coefficients will give good results.

The coefficients in the transformation equation (eqn. 12) are with one significant digit only. Pure pragmatic considerations are involved. In our case the transformation had to be used in a microprocessor electrocardiograph and the computation speed was important. So a search was performed for a 'one significant digit' solution with a reasonably reduced number of coefficients. The actual values of the coefficients were obtained after estimation tests on several possible transformations located near the original

$T3$ transformation. It can be seen in Table 2 that $T5$ is very close in performance to $T3$. Waveforms produced by $T3$ and $T5$ are very close and visually it is almost impossible to detect any difference.

4.8 Clinical tests

The recordings of all 92 individuals (Frank and $T5$ sets) were presented independently to two cardiologists in the Medical Academy. The diagnostic agreement between two sets was found to be quite satisfying but no quantitative estimations were made. A clinical test is in progress at the moment in two cardiological clinics. Frank and derived ($T5$) electrocardiograms are recorded in order to check whether Frank system diagnostic criteria might be used. The preliminary results are encouraging.

4.9 Comments on accuracy

The estimation of the transformation is given in terms of correlation coefficients and mean-squares distances. For a rough estimation one has to have in mind that beat-to-beat variability in an ECG signal is of the order of $r > 0.990$ and $d < 0.1$. A visual estimation is most important so that the full ECG data report on Fig. 5 might be of some help to the reader in deciding whether the results of the suggested derivation are successful.

5 Conclusion

It is unlikely that any orthogonal lead system should replace the conventional 12-lead system in the future. A great amount of 12-lead empirical clinical information has been collected and the availability of standard equipment is also to be considered.

On the other hand, the orthogonal lead systems have some advantages and the diagnostic accuracy will increase if both systems are used simultaneously. Recent developments in computer technology give new possibilities, and different ways of displaying the ECG signals can be implemented in one device as for instance, the conventional electrocardiograms and the vectorcardiographic loops. The ECG diagnostic programs also operate either with conventional or orthogonal signals or both. Thus, the proposed possibility of obtaining the conventional and orthogonal information from the same set of electrodes is important from a practical point of view.

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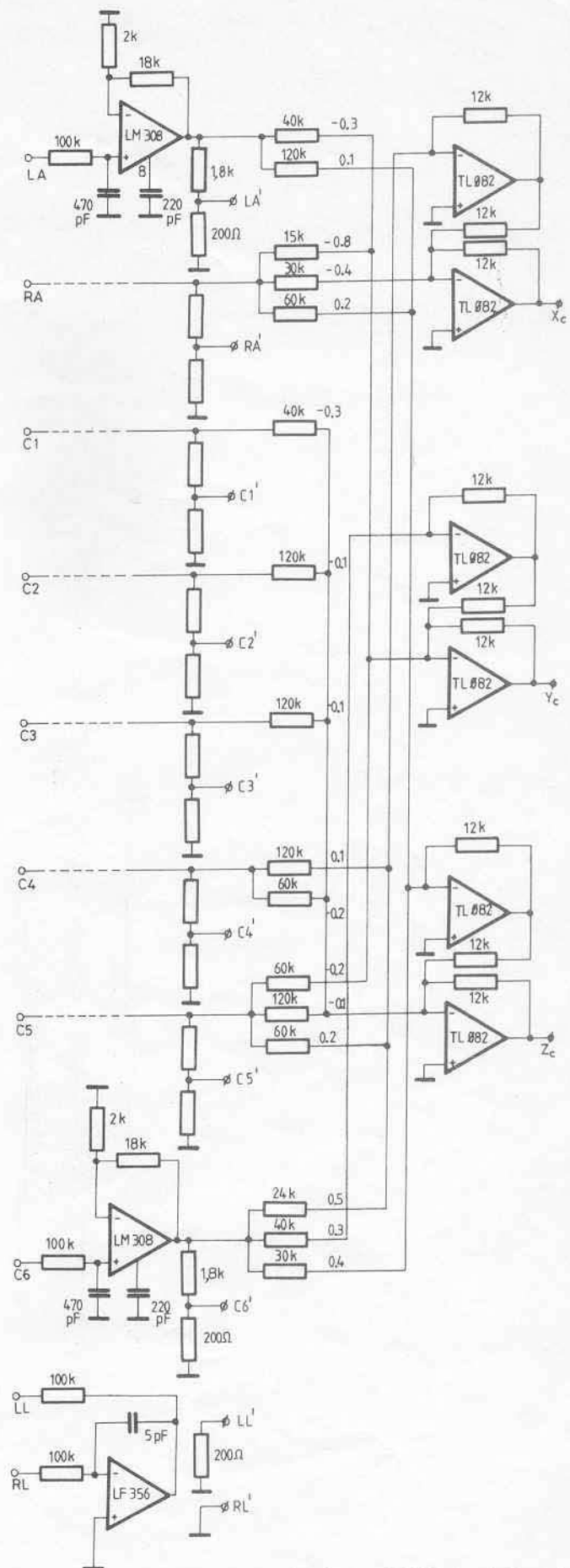


Fig. 6 Analogue computation of orthogonal XYZ signals from the conventional lead electrodes. Simultaneous recordings with a standard electrocardiograph and an orthogonal recording device are possible. The electrocardiograph patient cable is connected to the corresponding points LA' , RA' , $C1'$, $C2'$, $C3'$, $C4'$, $C5'$, $C6'$, LL' , RL' .